<u>Swarm Intelligence</u>

Ant Colony Optimization

Based on slides by Thomas Bäck, which were based on:
Marco Dorigo and Thomas Stützle: Ant Colony Optimization. MIT Press, Cambridge, MA, 2004.

Examples of Collective Intelligence in Nature

Termite hill

Nest of wasps

Bee attack

Flocking birds

Swarm Intelligence

- Originated from the study of colonies, or swarms of social organisms

- Collective intelligence arises from interactions among individuals having simple behavioral intelligence

- Each individual in a swarm behaves in a distributed way with a certain information exchange protocol

Communication

- **Point-to-point**: information between individuals or between an object and an individual is directly transferred
  - direct visual contact, antennation, trophallaxis (food or liquid exchange), chemical contact, …
- **Broadcast-like**: the signal propagates to some limited extent throughout the environment and/or is made available for a rather short time
  - generic visual detection, use of lateral line in fishes to detect water waves, actual radio broadcast
- **Indirect (stigmergy)**: two individuals interact indirectly when one of them modifies the environment and the other responds to the new environment at a later time
  - pheromone laying/following, post-it, web

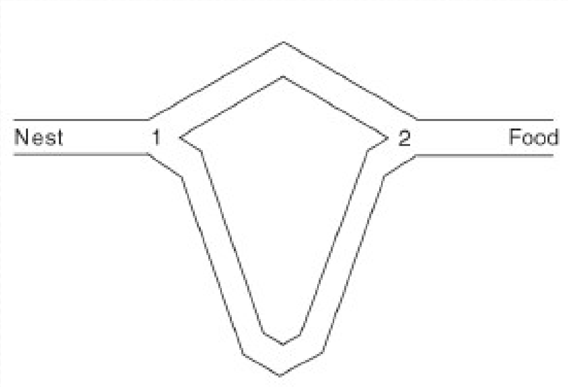
Ant Colony Optimisation

What is special about ants?

- Ants can perform complex tasks:
  - nest building, food storage
  - garbage collection, war
  - ***foraging*** *(to wander in search of food)*
- There is no management in an ant colony
  - collective intelligence
- They communicate using ***pheromones*** *(chemical substances)*, sound, touch
- Curiosities:
  - Ant colonies exist for more than 100 million years
  - Myrmercologists estimate that there are around 20 000 species of ants

Double Bridge Experiments

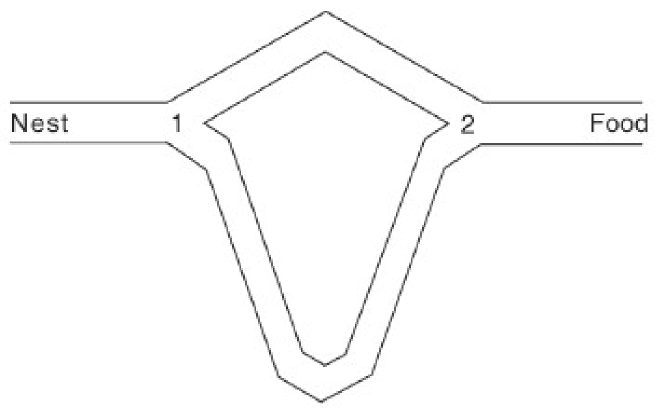
- A study on the pheromone trail-laying and –following behavior of Argentine ants
- A double bridge connects a nest of ants and a food source
- The ratio $r = L_{long} / L_{short}$ between the length of the two branches of the double bridge is varied
- Ants are free to move between the nest and the food

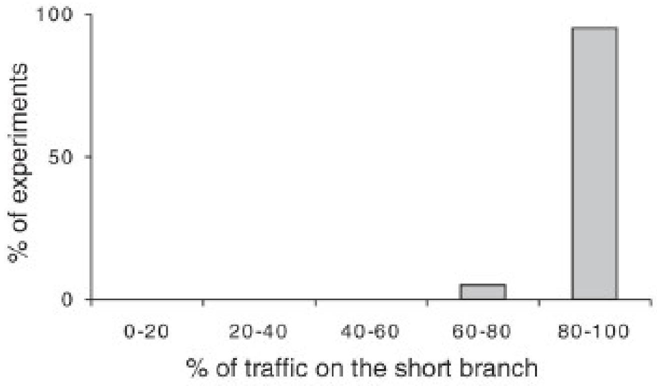


J.L. Deneubourg, S. Aron, S. Goss and J.M. Pasteels (1990). The self-organizing exploratory pattern of the Argentine ant. Journal of Insect Behaviour, 3, 159-168.

S. Goss, S. Aron, J.L. Deneubourg and J.M. Pasteels (1989). Self-organized shortcuts of the Argentine ant. Naturwissenschaften, 76, 579-581

Double Bridge Experiments





- In most of the trials, almost all the ants select the short branch (exploitation)
- Not all ants use the short branch, but a small percentage may take the longer one (exploration)

Foraging Behavior of Argentine Ants

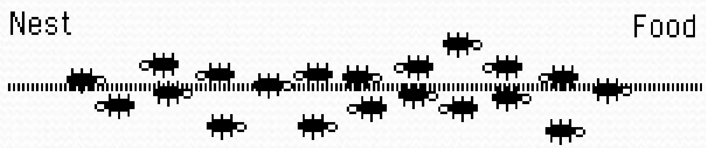
- Ants initially explore the area surrounding their nest randomly
- Argentinian ants deposit pheromones everywhere they go
- When choosing their way, ants prefer to follow strong pheromone concentrations
- Pheromones defuse over time

Foraging Behavior of Argentine Ants

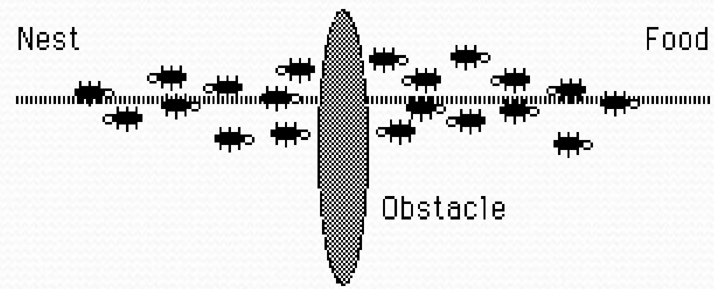
- How do Argentine ants find the shortest path?
  - The ants that take the shortest path arrive at the food source first
  - They return over the path that they took to get there, reinforcing the pheromones they deposited when going to the food source
  - Other ants notice the trail and follow it, reinforcing it further
- Hence, during the "start" of the experiment the advantage that ants on the shortest path had is reinforced

Alternative experiment

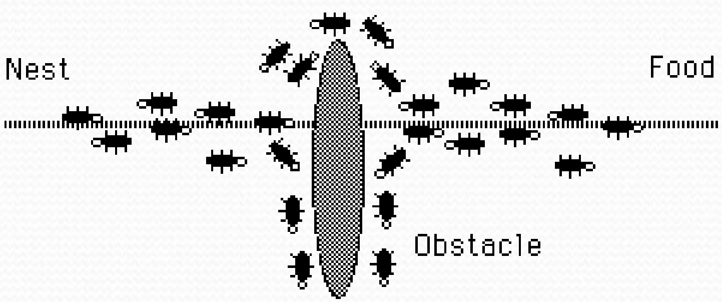
- An obstacle is put in the path of ants



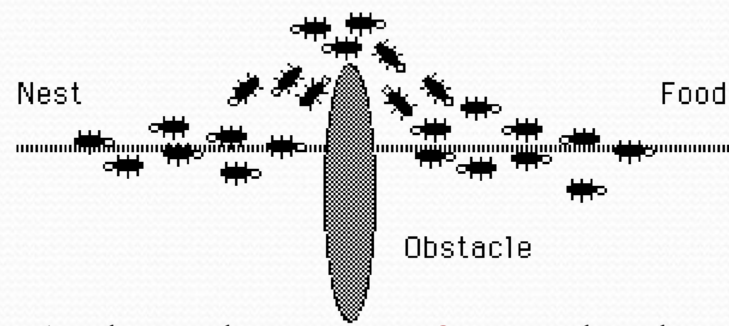
a) - Ants **follow path** between the Nest and the Food Source



b) - Ants go around the obstacle following one of two different paths with **equal probability**



c) Ants on the shortest path arrives at the food source first; on the way back they will follow the phero-mones on the shortest path again



d) – At the end, **all ants follow** the shortest path.

Simple Ant Colony Optimisation: Shortest Paths

- Artificial ants going "forward"
  - choose probabilistically the next node on their path, exploiting pheromones
  - do not drop pheromones
  - memorize the path they take
- Artificial ants going "backward"
  - deterministically follow the path they took earlier
  - drop pheromones proportionally to the quality of the path taken earlier

Simple ACO: Shortest Paths

```
initialize pheromones
for each iteration do
    for k = 1 to number of ants do
        set out ant k at start node
        while ant k has not build a solution do
            choose the next node of the path
        end while
    end for
    update pheromones
end for
return best solution found
```

Simple ACO: Shortest Paths

- For an ant located at node v_i the probability p_{ij} of choosing v_j as the next node is:

$$p_{ij}^k = \begin{cases} \frac{(\tau_{ij})^\alpha}{\sum_{m \in N_i^k} (\tau_{im})^\alpha} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases}$$

where

  - τ_{ij} is the amount of pheromones on edge $i \rightarrow j$
  - N_i^k is the set of neighbors of node i not visited by ant k yet (tabu list)

Simple ACO: Shortest Paths

- Change in pheromone for an ant k on edge i → j

$$\Delta\tau_{ij}^{k} = \begin{cases} Q/L_k & \text{if } (i,j) \in T_k \\ 0 & \text{otherwise} \end{cases}$$

where:

- Q : a heuristic parameter
- T_k : the path traversed by ant k
- L_k : the length of T_k calculated as the sum of all lengths of edges in T_k

Simple ACO: Shortest Paths

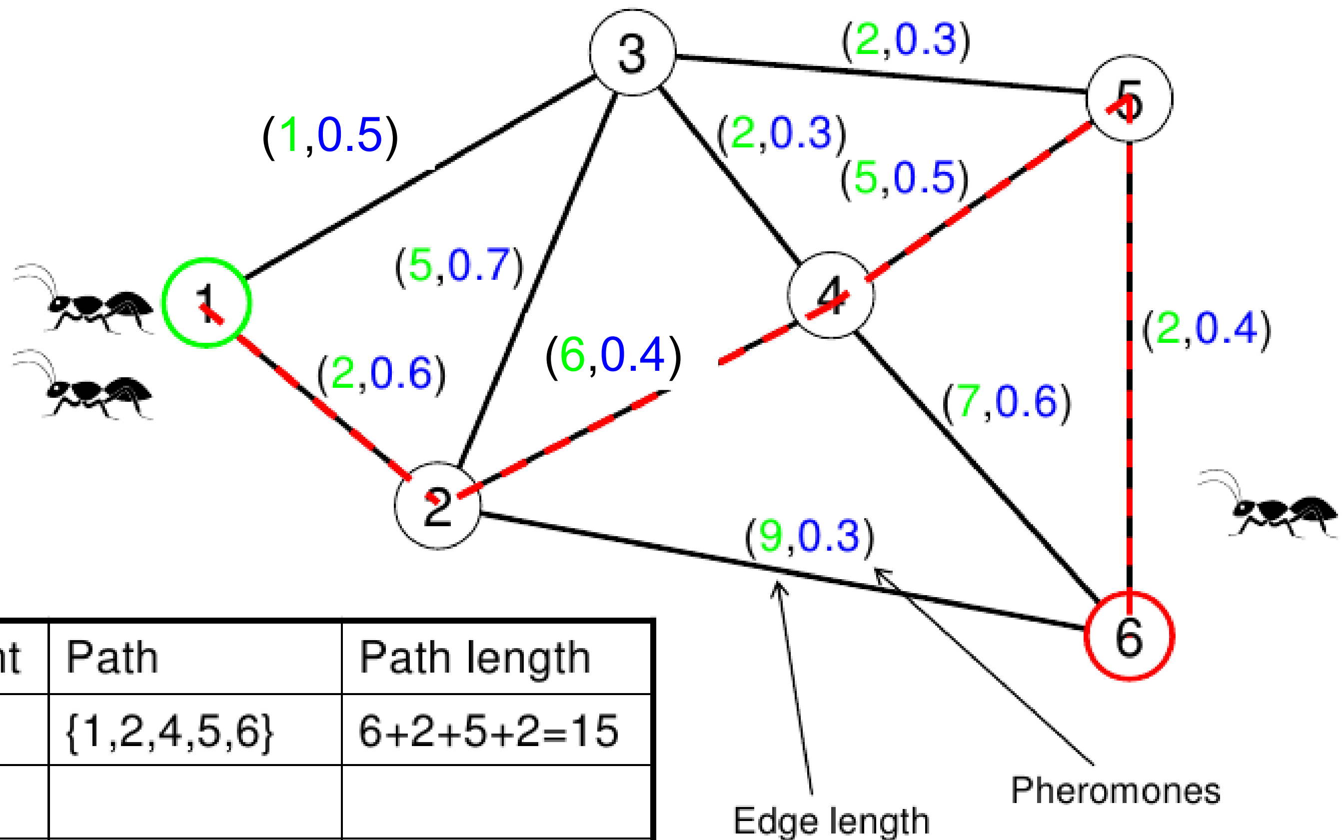
- Pheromone update on an edge i → j

$$\tau_{ij} = (1-\rho)\tau_{ij} + \sum_{k=0}^{m} \Delta\tau_{ij}^{k}$$

with

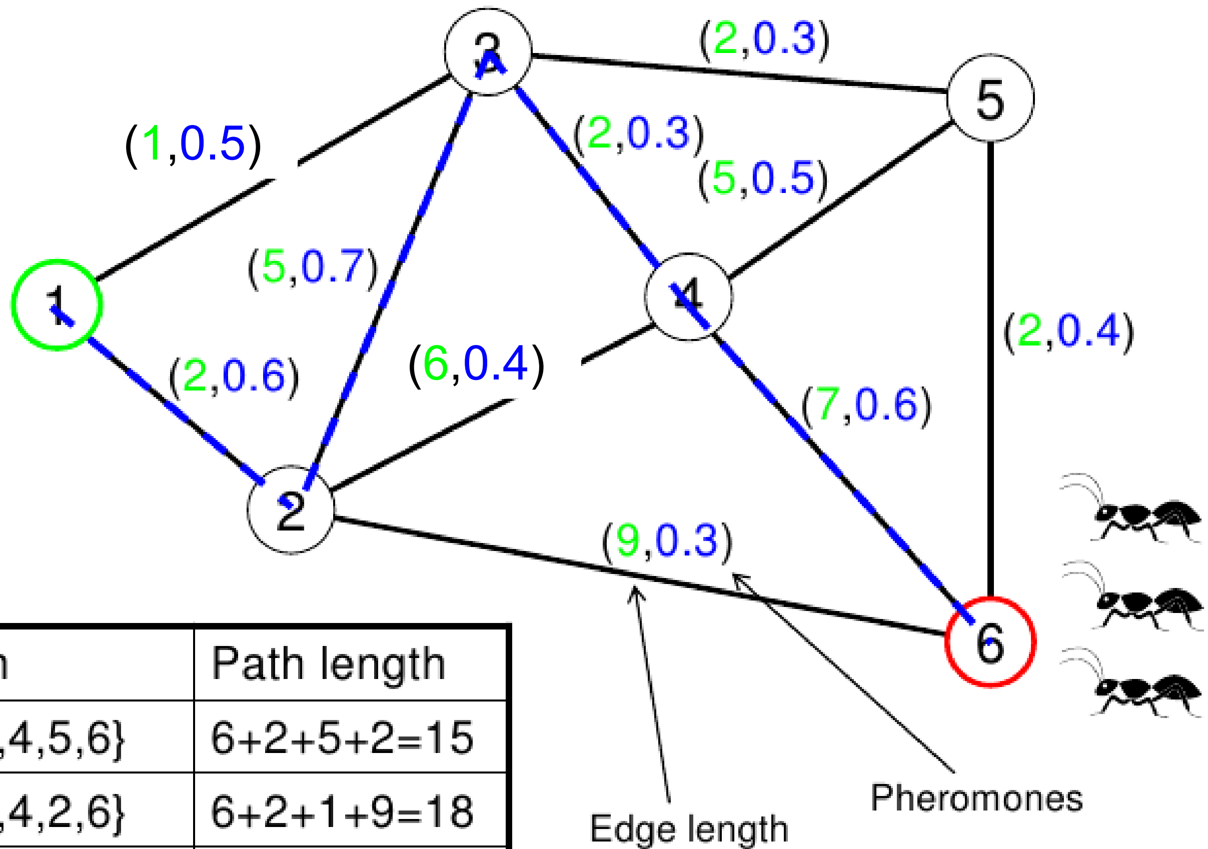
- ρ : the evaporation rate of the old pheromone

Simple ACO: Shortest Paths



Ant	Path	Path length
1	{1,2,4,5,6}	6+2+5+2=15
2		
3		

Simple ACO: Shortest Paths



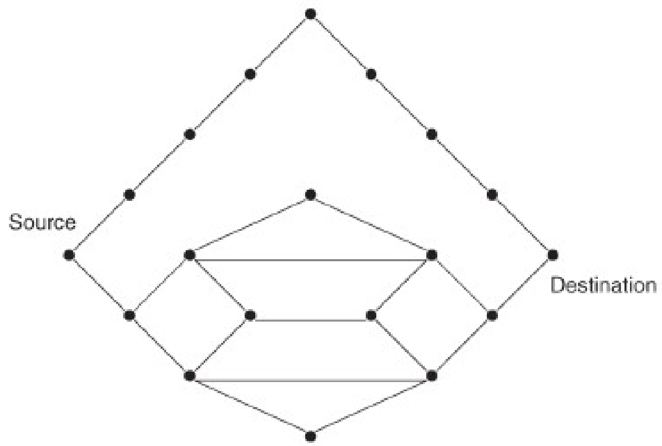
Ant	Path	Path length
1	{1,2,4,5,6}	6+2+5+2=15
2	{1,3,4,2,6}	6+2+1+9=18
3	{1,2,3,4,6}	2+5+2+7=16

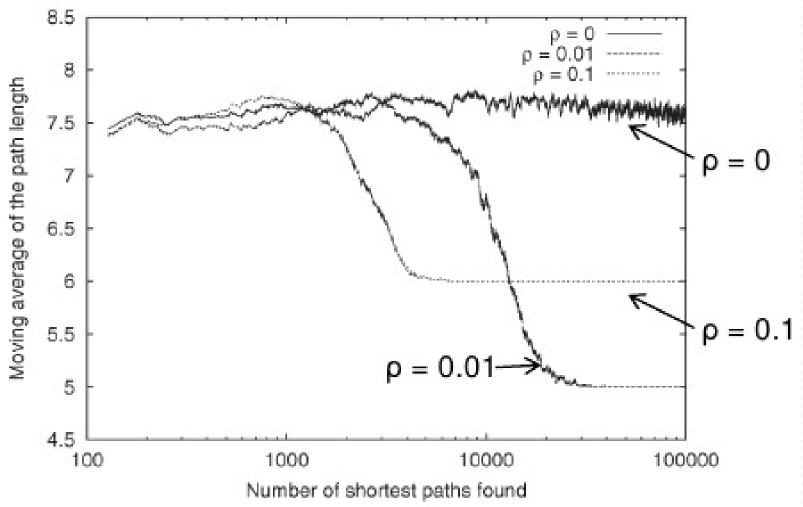
Simple ACO: Shortest Paths

$Q = 1, \rho = 0.1$

	τ_{old}	$\Delta\tau_{ij}^1$	$\Delta\tau_{ij}^2$	$\Delta\tau_{ij}^3$	$\Delta\tau_{ij}$	τ_{new}
(1,2)	0.6	1/15	0	1/16	1/15 + 1/16 ≈ 0.129	0.6 * 0.9 + 0.129 = 0.669
(1,3)	0.5	0	1/18	0	1/18 ≈ 0.055	0.5 * 0.9 + 0.055 = 0.505
(2,3)	0.7	0	0	1/16	1/16 ≈ 0.063	0.7 * 0.9 + 0.063 = 0.693
(2,4)	0.4	1/15	1/18	0	1/15 + 1/18 ≈ 0.122	0.4 * 0.9 + 0.122 = 0.482
(2,6)	0.3	0	1/18	0	1/18 ≈ 0.055	0.3 * 0.9 + 0.055 = 0.325
(3,4)	0.3	0	1/18	1/16	1/18 + 1/16 ≈ 0.118	0.3 * 0.9 + 0.118 = 0.388
(3,5)	0.3	0	0	0	0	0.3 * 0.9 + 0 = 0.27
(4,5)	0.5	1/15	0	0	1/15 ≈ 0.067	0.5 * 0.9 + 0.067 = 0.517
(4,6)	0.6	0	0	1/16	1/16 ≈ 0.063	0.6 * 0.9 + 0.063 = 0.603
(5,6)	0.4	1/15	0	0	1/15 ≈ 0.067	0.4 * 0.9 + 0.067 = 0.427

Simple ACO: Shortest Paths





- Low ρ → low evaporation → slow convergence, “old” paths continue to be traversed instead of searching new ones
- High ρ → high evaporation → very fast convergence, but due to limited memory no drive to explore variations of a good path

Ant Systems for the Traveling Salesman Problem

- The first ACO algorithm proposed by Dorigo et al. in 1991

```
procedure Ant System for TSP
        Pheromone Initialization
        while (not terminate) do
                for i = 1 to k do
                        Tour Construction
                end
                Update Pheromones
        end
end
```

Ants for TSP

- For an ant located at node v_i the probability p_{ij} of choosing v_j as the next node is:

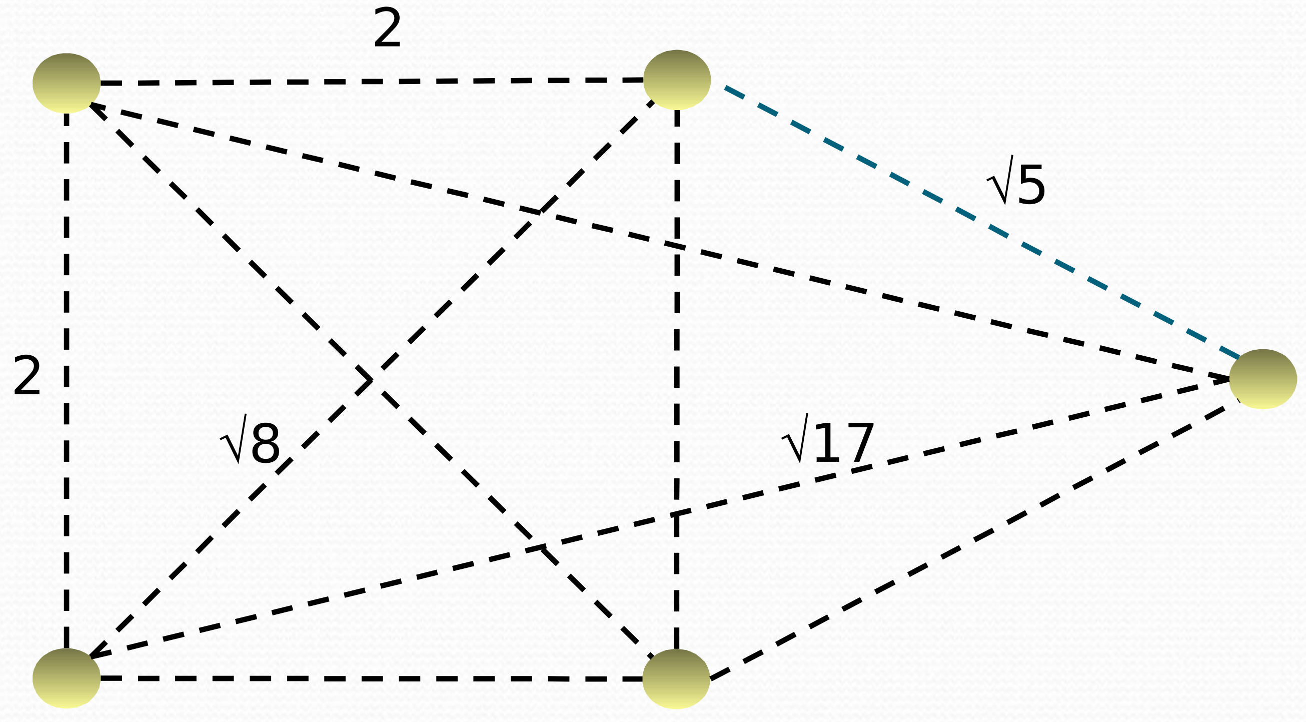
$$p_{ij}^k = \begin{cases} \frac{(\tau_{ij})^\alpha (\eta_{ij})^\beta}{\sum_{m \in N_i^k} (\tau_{im})^\alpha (\eta_{ij})^\beta} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases}$$

where

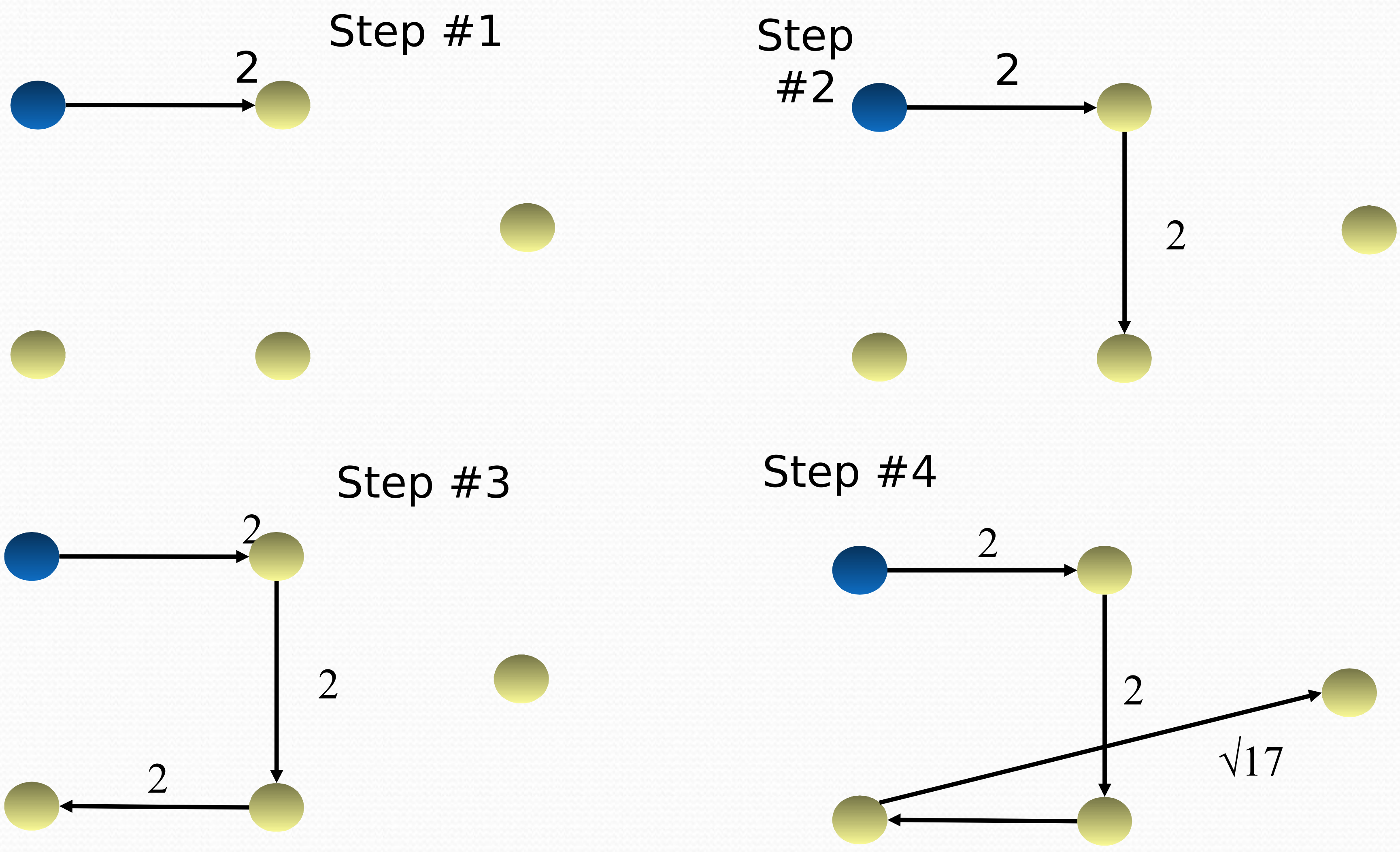
- τ_{ij} is the amount of pheromones on edge $i \rightarrow j$
- N_i^k is the set of neighbors of node i not visited by ant k yet (tabu list)
- η_{ij} is the heuristic desirability of the edge (i.e. 1 / distance between nodes)

Traveling Salesman Problem

- *n* cities (5)
- Number of possible paths: $(n-1)! / 2$

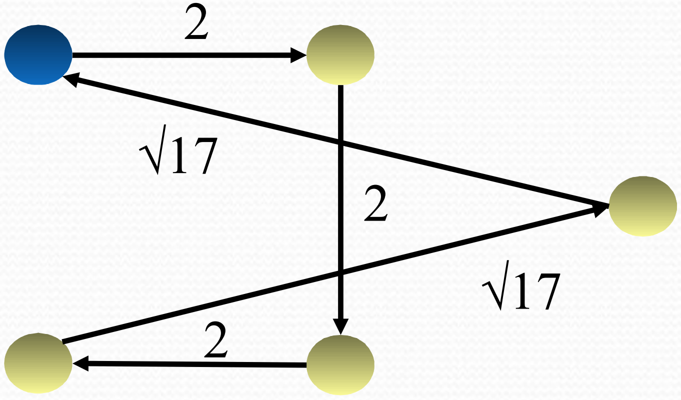


Solution using "nearest city" heuristic

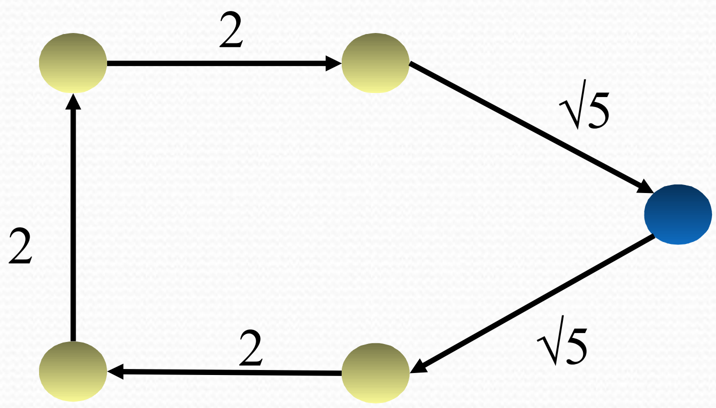


Solution using "nearest city" heuristic

- The final solution is obviously non-optimal

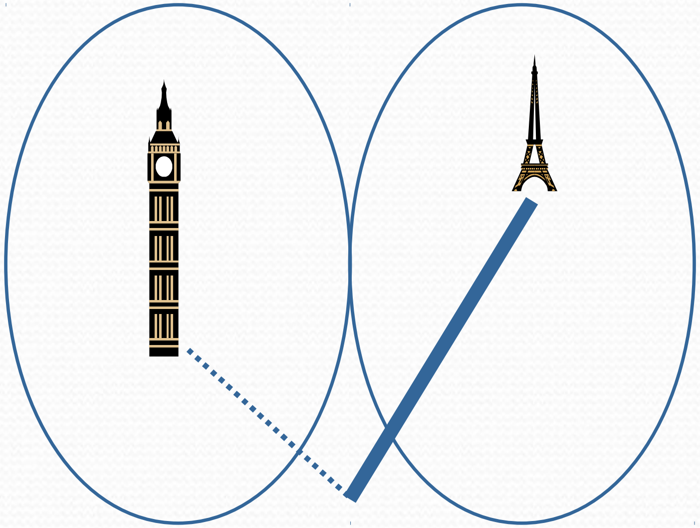


- This heuristic can give the optimal solution if it is given a proper initial node



ACO in Travelling Salesman Problem

- m ants
- n cities
- $\eta = 1 / d$

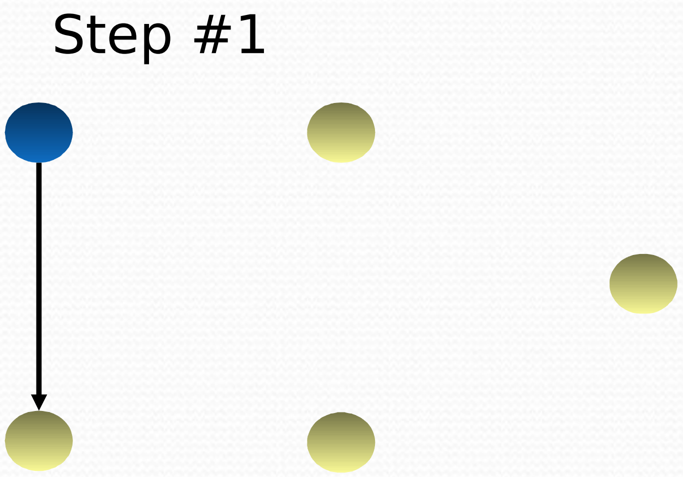


η vs τ

The ACO balances the heuristic information with the experience (pheromone) information

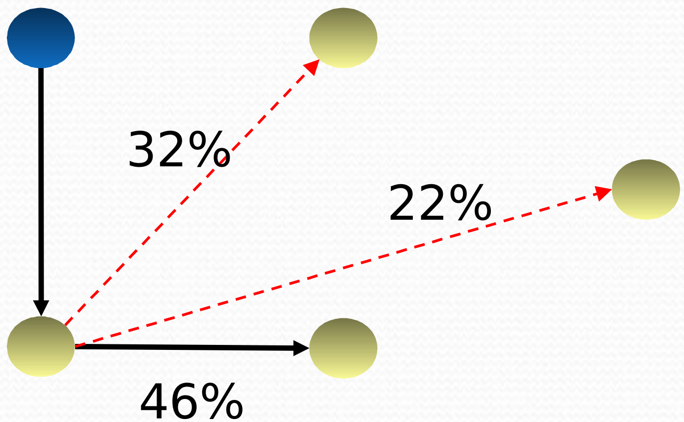
Iteration i=1, Ant m=1

- All paths have the same pheromone intensity τ_0=0.5
- Pheromone trail and heuristic information have the same weight $\alpha = 1$, $\beta = 1$, ρ=0.1
- An ant is randomly placed
- The probability to choose is, in this case, based only on heuristic information
  - P_{12}=31%
  - P_{13}=16%
  - P_{14}=22%
  - P_{15}=31%
- Ant m = 1 chooses node 5

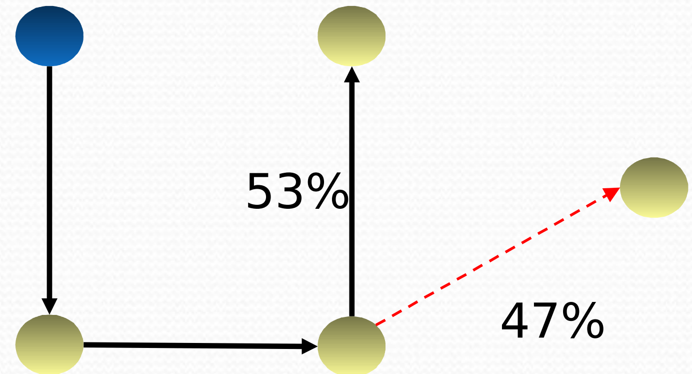


Iteration i=1, Ant m=1

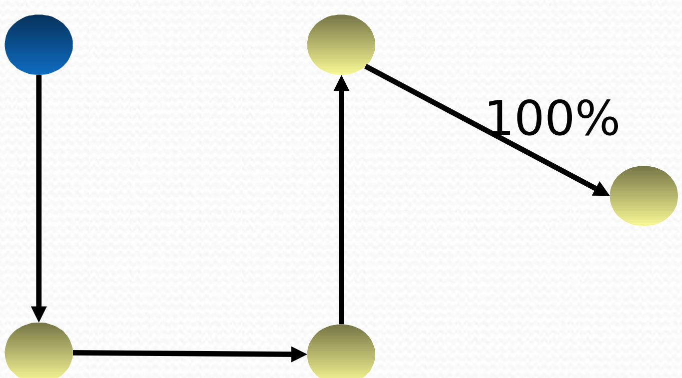
Step #2



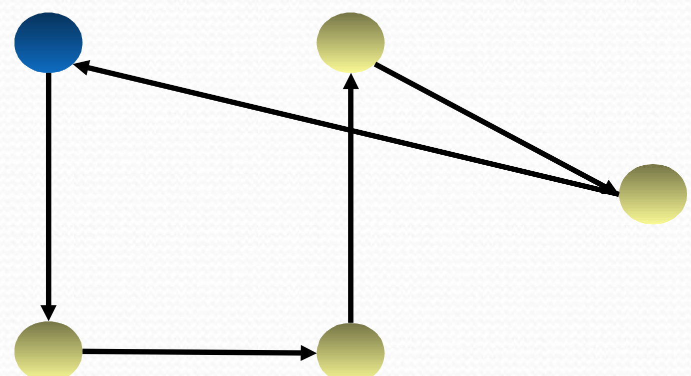
Step #3



Step #4



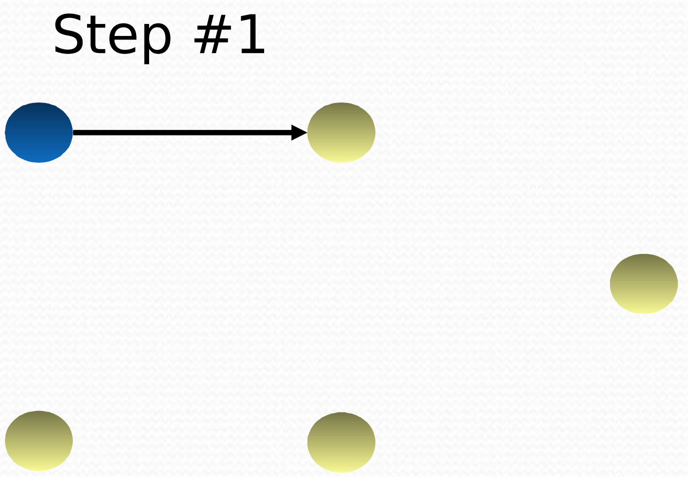
Step #5



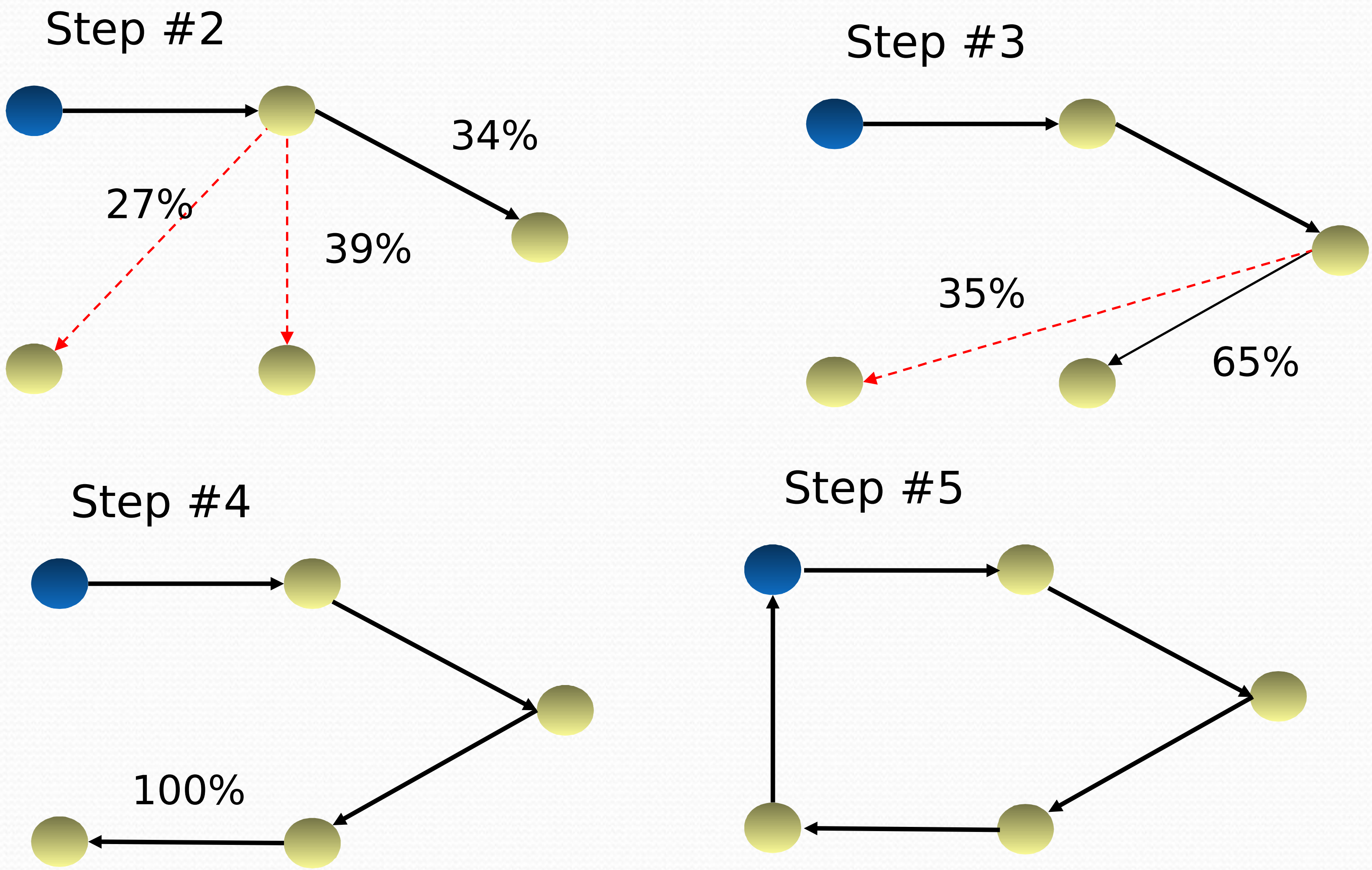
$$f_1 = 2 + 2 + 2 + \sqrt{5} + \sqrt{17} = 12.36$$

Iteration i=1, Ant m=2

- All paths have the same pheromone intensity $\tau_0=0.5$
- Pheromone trail and heuristic information have the same weight $\alpha = 1$, $\beta = 1$, $\rho=0.1$
- An ant is randomly placed
- The probability to choose is, in this case, based only on heuristic information
  - P12=31%
  - P13=16%
  - P14=22%
  - P15=31%
- Ant m = 2 chooses node 2



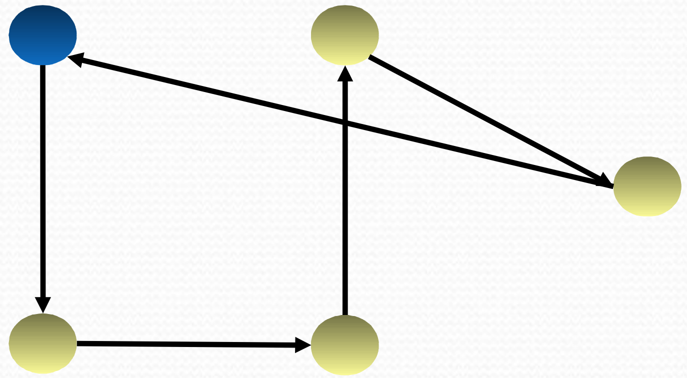
Iteration i=1, Ant m=2



$$f_2 = 2 + \sqrt{5} + \sqrt{5} + 2 + 2 = 10.47$$

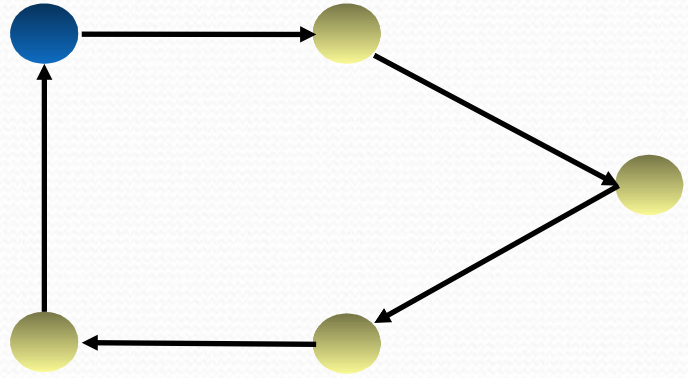
Iteration i=1, Pheromone Update

- The final solution of ant m=1 is D=12.36. The reinforcement produced by this ant m=1 is 0,081.



$Q = 1, \mu = Q/D$

- The final solution of ant m=2 is D=10,47. The reinforcement produced by ant m=2 is 0,095!



Updating Pheromone Matrix

- The pheromone update can be done following different approaches:
  - Considering the pheromone dropped by every ant

$$\tau(l+1)=\begin{bmatrix}0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\end{bmatrix}\times(1-\rho)+\begin{bmatrix}0 & 0 & 0 & 0 & 0.08\\ 0 & 0 & 0.08 & 0 & 0\\ 0.08 & 0 & 0 & 0 & 0\\ 0 & 0.08 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.08 & 0\end{bmatrix}+\begin{bmatrix}0 & 0.095 & 0 & 0 & 0\\ 0 & 0 & 0.095 & 0 & 0\\ 0 & 0 & 0 & 0.095 & 0\\ 0 & 0 & 0 & 0 & 00.95\\ 0.095 & 0 & 0 & 0 & 0\end{bmatrix}$$

  - Considering the pheromone dropped by the best ant of the present iteration

$$\tau(l+1)=\begin{bmatrix}0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\end{bmatrix}\times(1-\rho)+\begin{bmatrix}0 & 0.095 & 0 & 0 & 0\\ 0 & 0 & 0.095 & 0 & 0\\ 0 & 0 & 0 & 0.095 & 0\\ 0 & 0 & 0 & 0 & 00.95\\ 0.095 & 0 & 0 & 0 & 0\end{bmatrix}$$

  - Considering the pheromone dropped by the best ant in all iterations (after iteration N=1, this is the same as the previous approach)

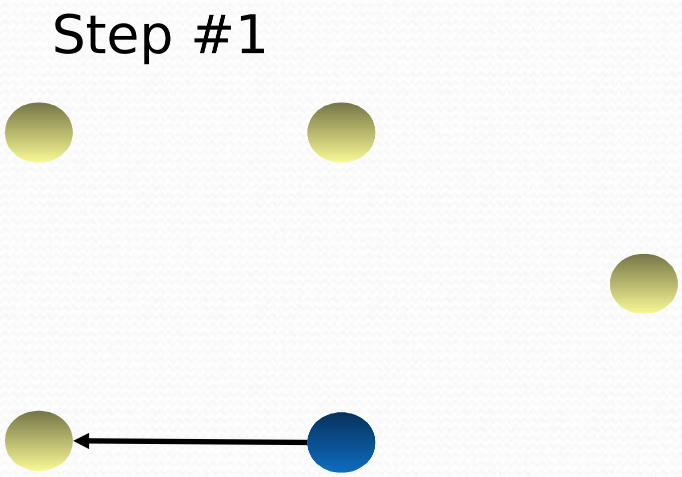
Updating Pheromone Matrix

- Update the pheromone on each edge by:

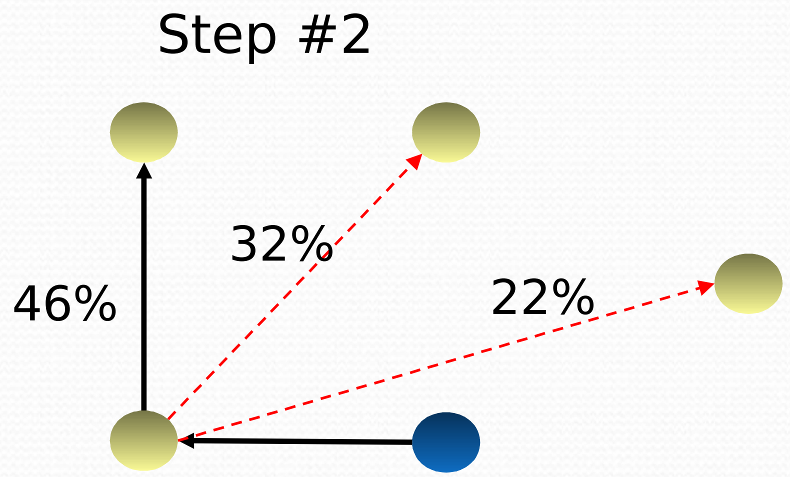
$$\tau(l+1)=\begin{bmatrix}0.5&0.5&0.5&0.5&0.5\\0.5&0.5&0.5&0.5&0.5\\0.5&0.5&0.5&0.5&0.5\\0.5&0.5&0.5&0.5&0.5\\0.5&0.5&0.5&0.5&0.5\end{bmatrix}\times(1-\rho)+\begin{bmatrix}0&0&0&0&0.08\\0&0&0.08&0&0\\0.08&0&0&0&0\\0&0.08&0&0&0\\0&0&0&0.08&0\end{bmatrix}+\begin{bmatrix}0&0.095&0&0&0\\0&0&0.095&0&0\\0&0&0&0.095&0\\0&0&0&0&00.95\\0.095&0&0&0&0\end{bmatrix}$$

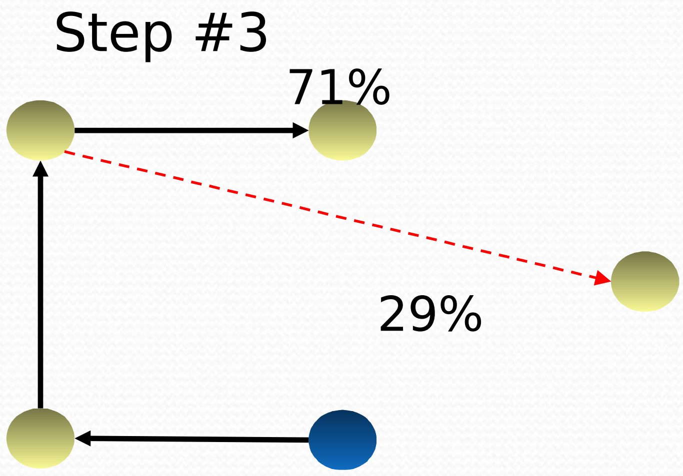
Iteration i=2, Ant m=1

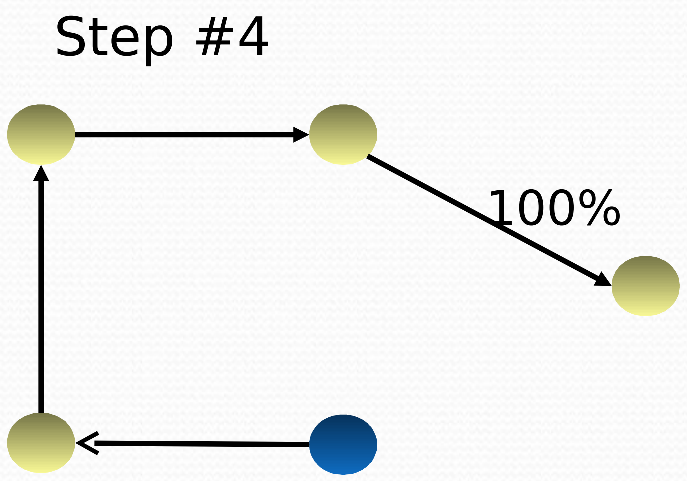
- The pheromone trails have different intensities
- Pheromone trail and heuristic information have the same weight $\alpha = 1$, $\beta = 1$, $\rho=0.1$
- An ant is randomly placed
- The probability to choose is
  - P_{41}=19%
  - P_{42}=26%
  - P_{43}=23%
  - P_{45}=32%
- Ant m = 1 chooses node 5

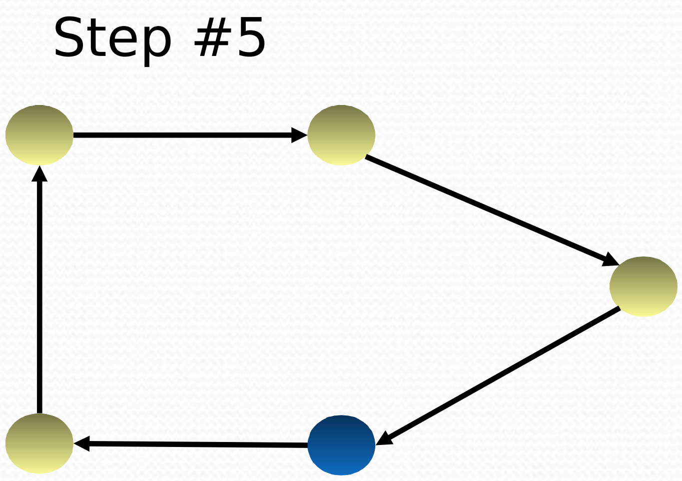


Iteration i=2, Ant m=1





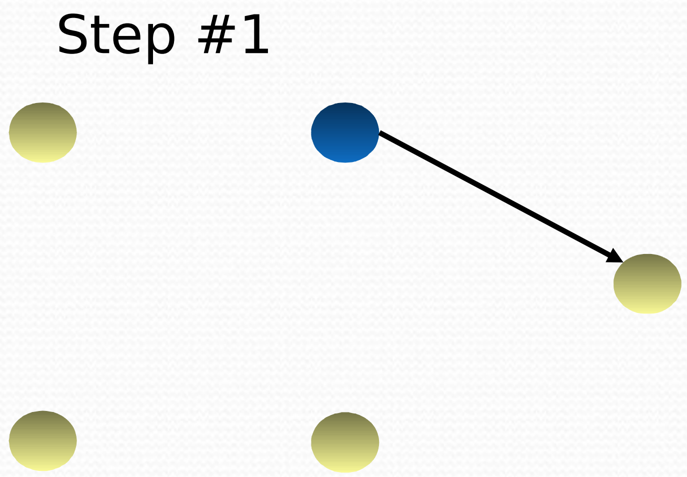




$$f_1 = 2 + 2 + 2 + \sqrt{5} + \sqrt{5} = 10.47$$

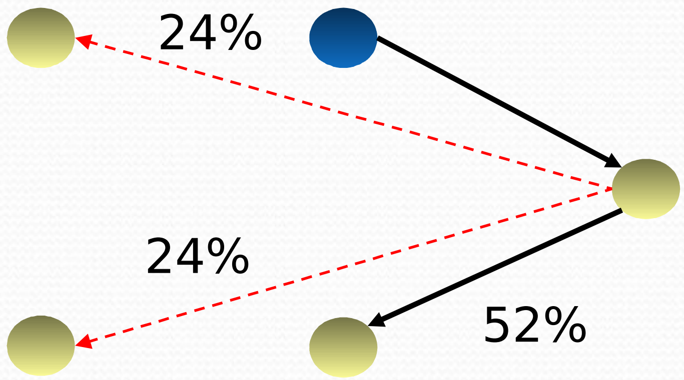
Iteration i=2, Ant m=2

- The pheromone trails have different intensities
- Pheromone trail and heuristic information have the same weight $\alpha = 1$, $\beta = 1$, $\rho=0.1$
- An ant is randomly placed
- The probability to choose is
  - $P_{21}=26\%$
  - $P_{23}=29\%$
  - $P_{24}=26\%$
  - $P_{25}=19\%$
- Ant m = 2 chooses node 3

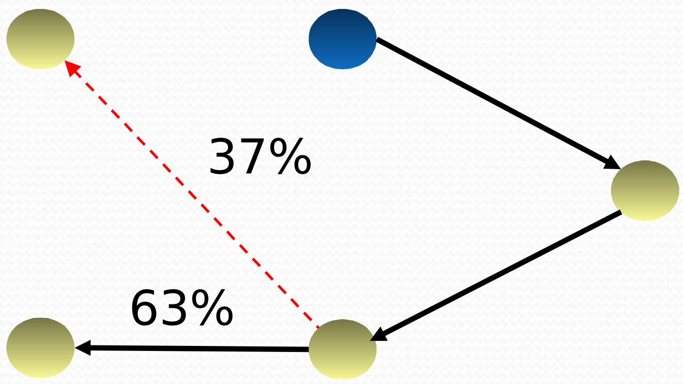


Iteration i=2, Ant m=2

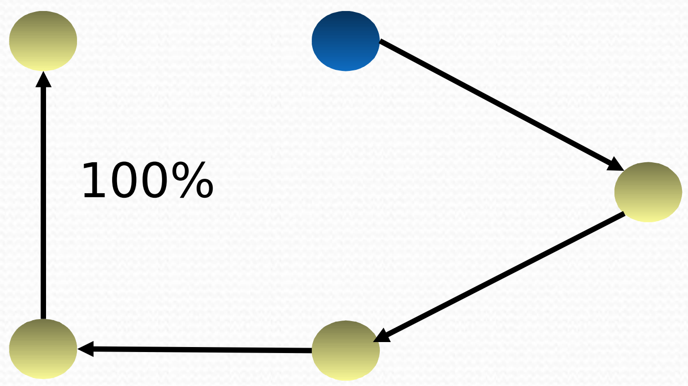
Step #2



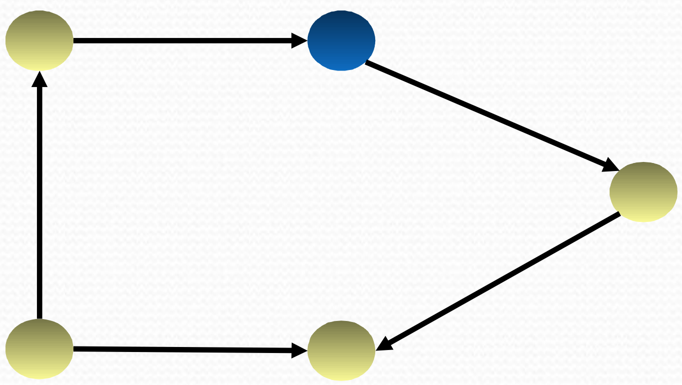
Step #3



Step #4



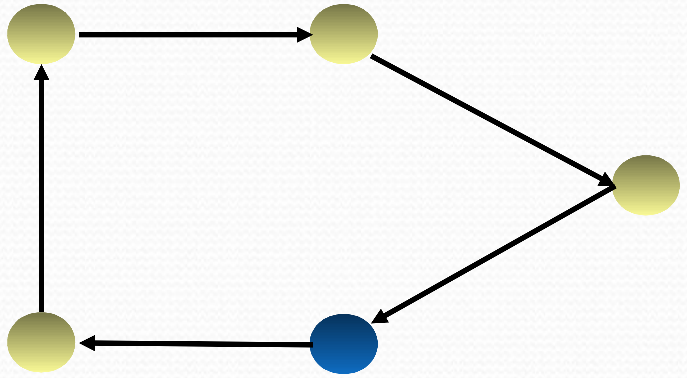
Step #5

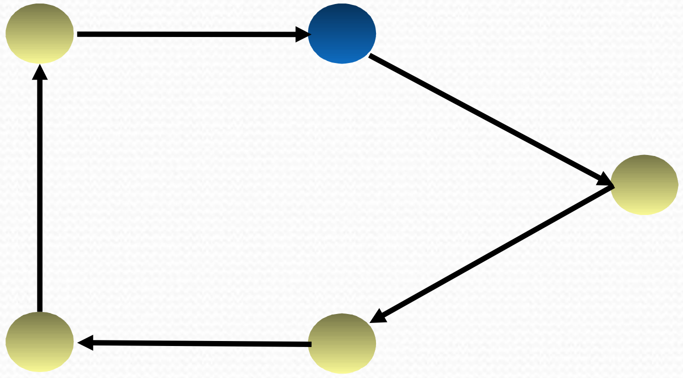


$$f_2=\sqrt{5}+\sqrt{5}+2+2+2=10.47$$

Iteration i=2, Pheromone Update

- The final solution of ant m=1 and m=2 is D=10,47. The reinforcement produced by each ant is 0,095!





Updating Pheromone Matrix

- Considering the pheromone dropped by every ant

$$\tau(l+1)=\begin{bmatrix}0.45 & 0.55 & 0.45 & 0.45 & 0.53\\ 0.45 & 0.45 & 0.63 & 0.45 & 0.45\\ 0.53 & 0.45 & 0.45 & 0.55 & 0.45\\ 0.45 & 0.53 & 0.45 & 0.45 & 0.55\\ 0.55 & 0.45 & 0.45 & 0.53 & 0.45\end{bmatrix}\times(1-\rho)+\begin{bmatrix}0 & 0.095 & 0 & 0 & 0\\ 0 & 0 & 0.095 & 0 & 0\\ 0 & 0 & 0 & 0.095 & 0\\ 0 & 0 & 0 & 0 & 00.95\\ 0.095 & 0 & 0 & 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0.095 & 0 & 0 & 0\\ 0 & 0 & 0.095 & 0 & 0\\ 0 & 0 & 0 & 0.095 & 0\\ 0 & 0 & 0 & 0 & 00.95\\ 0.095 & 0 & 0 & 0 & 0\end{bmatrix}$$

Ant System for TSP

1. Initialize:
   Set $t := 0$ $\{t$ is the time counter$\}$
   Set NC $:= 0$ {NC is the cycles counter}
   For every edge (i, j) set an initial value $\tau_{ij}(t) = c$ for trail intensity and $\Delta\tau_{ij} = 0$
   Place the m ants on the n nodes

2. Set $s := 1$ $\{s$ is the tabu list index$\}$
   For $k := 1$ to m do
   Place the starting town of the kth ant in $\mathbf{tabu}_k(s)$

3. Repeat until tabu list is full
   {this step will be repeated $(n-1)$ times}
   Set $s := s + 1$
   For $k := 1$ to m do
   Choose the town j to move to, with probability $p_{ij}^k(t)$ given by Eq. (4)
   {at time t the kth ant is on town $i = \mathbf{tabu}_k(s-1)$}
   Move the kth ant to the town j
   Insert town j in $\mathbf{tabu}_k(s)$

4. For $k := 1$ to m do
   Move the kth ant from $\mathbf{tabu}_k(n)$ to $\mathbf{tabu}_k(1)$
   Compute the length L_k of the tour described by the kth ant
   Update the shortest tour found
   For every edge (i, j)
   For $k := 1$ to m do

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{L_k} & \text{if } (i,j) \in \text{tour described by } \mathbf{tabu}_k \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta\tau_{ij} := \Delta\tau_{ij} + \Delta\tau_{ij}^k;$$

5. For every edge (i, j) compute $\tau_{ij}(t+n)$ according to equation $\tau_{ij}(t+n) = \rho \cdot \tau_{ij}(t) + \Delta\tau_{ij}$
   Set $t := t + n$
   Set NC $:=$ NC $+ 1$
   For every edge (i, j) set $\Delta\tau_{ij} := 0$

6. If (NC $<$ NC_{MAX}) and (not stagnation behavior) then
   Empty all tabu lists
   Goto step 2
   else
   Print shortest tour
   Stop

ACO General Framework

Initialize pheromones

while termination conditions not met **do**

  Construct ant solutions based on the pheromones

  Update pheromones

  Perform daemon actions (optional)

end while

Additional local search to improve solutions often necessary

ACO Variations

- Elitist Ant System (EAS)
- Rank-Based Ant System (ASrank)
- Min-Max Ant System (MMAS)
- Ant Colony System (ACS)

Elitist Ant Systems

- Modification of the ant system in which the best-so-far solution is reinforced additionally:

$$\tau_{ij} = (1-\rho)\tau_{ij} + \sum_{k=0}^{m} \Delta\tau_{ij}^{k} + e \cdot \Delta\tau_{ij}^{bs}$$

With

- *bs:* the best solution found till now
- *e*: parameter for the contribution of the best solution

Rank-based Ant Systems

- Each ant deposits pheromone proportional to its rank in the set of ants
- Only the best *e* solutions deposit pheromones
- The best-so-far solution deposits most pheromones

$$\tau_{ij} = (1-\rho)\tau_{ij} + \sum_{k=1}^{e-1}(e-k)\Delta\tau_{ij}^{k} + e \cdot \Delta\tau_{ij}^{bs}$$

where it is assumed that the ants are ordered in the order of the quality of the solution they represent

MAX-MIN Ant Systems

- Ignore all ants, except the best ant in its iteration:

$$\tau_{ij} = (1 - \rho)\tau_{ij} + \Delta\tau_{ij}^{best}$$

where *best* is the best ant in the iteration

- To avoid stagnation:
  - limit the range of pheromone levels on edges

$$\tau_{min} \leq \tau_{ij} \leq \tau_{max}$$

  - if no better solutions is found after a number of iterations, reinitialize the pheromone levels to the upperbound

Ant Colony Systems

- New rule for picking a neighbor during the ant's traversal of a graph

$$j = \begin{cases} \arg\max_{l \in N_i^k} \tau_{il} (\eta_{il})^\beta & \text{if } q \leq q_0 \\ J & \text{otherwise} \end{cases}$$

with

  - q is a number drawn from a uniform distribution
  - q_0 is a parameter indicating the likelihood of making the choice to use the new selection criterion
  - J is the choice made using the traditional probabilitic selection

Ant Colony Systems

- Update pheromones
  - **after** all ants have traversed the graph:
    using only the 'best' ant

$$\tau_{ij} = (1-\rho)\tau_{ij} + \rho\Delta\tau_{ij}^{best}$$

  - **while** an ant is traversing the graph, for each edge that is passes:

$$\tau_{ij} = (1-\varphi)\cdot\tau_{ij} + \varphi\cdot\tau_0$$

→ encourage other ants in the same iteration to use a different path

Advantages / Disadvantages

- Advantages:
  - Applicable to a broad range of optimization problems
  - Can be used in dynamic applications (adapts to changes such as new distances, etc.)
  - Can compete with other global optimization techniques like genetic algorithms and simulated annealing
- Disadvantages:
  - Mainly applicable for discrete problems
  - Theoretical analysis is difficult

Example: Bankruptcy Prediction

- Bankruptcy prediction is a classification problem:

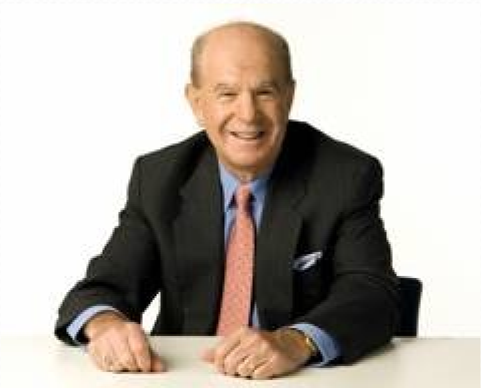
  find a classification rule that will separate firms that will go bankrupt from those that will not

- The set of attributes is usually a set of financial variables

- Most successful breakthrough in BP by Altman, 1968

Bankruptcy Prediction

- Altman selected in first instance 5 variables out of a list of 22 financial variables.



X_1: Working Capital / Total Assets

X_2: Retained Earnings / Total Assets

X_3: EBIT / Total Assets

X_4: MV of Equity / BV of Debt

X_5: Sales / Total Assets

$$Z = .012X_1 + .014X_2 + .033X_3 + .006X_4 + .999X_5$$

Altman's Data

- The dataset used by Altman consisted of 66 companies, 33 bankrupt (B) and 33 non bankrupt (NB)

Bankrupt
- Asset size between 0.6 mil. and 25.9 mil.
- Filed for bankruptcy between 1946 – 1965
- Using data for the 5 variables from 1 year before filing for bankruptcy

Non-Bankrupt
- Asset size between 1 mil. and 25 mil.
- Still in existence in 1966

Formalisation as Discrete Optimisation Problem

- For each variable (attribute) in the analysis, we generate cutpoints to discretise the data

- All possible cutpoints for a variable Xi are obtained by dividing the interval [min(i), max(i)] into a fixed number of smaller intervals

  → For each variable *i* we have cutpoints *j*, θ_{ij}

- For each variable *i* we have to choose one θ_{ij}

Formalisation as Discrete Optimisation Problem

- Evaluation of a choice of cutpoints:
  - we predict bankruptcy for a firm *k* with attributes

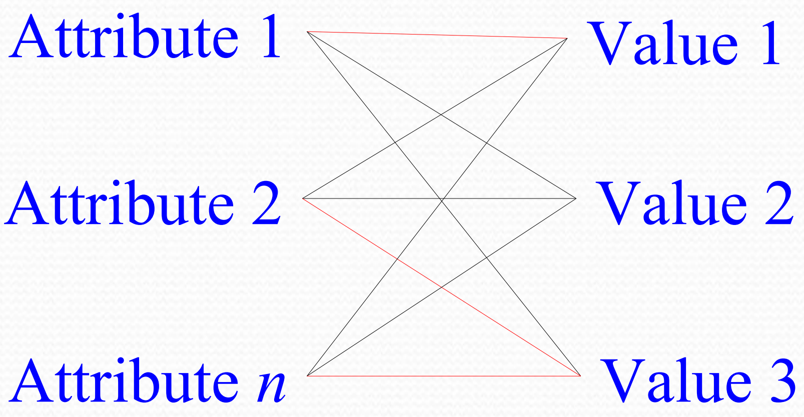
    $\xi_1^k, \ldots \xi_n^k$

    if $\xi_1^k \leq \theta_{1c(1)} \wedge \cdots \wedge \xi_1^k \leq \theta_{nc(n)}$

    where *c(i)* is the cutpoint chosen for attribute *i*
  - quality of a solution: the error of this choice of cutpoints on the training data

Ant Optimisation Representation

- We can see assignments as a choice of edges in a bipartite graph → update pheromones for each edge



Ant Optimisation Representation

- Pheromone update for **ant** *k*

$$\Delta_{ij}^{k} = \begin{cases} A & \text{if } c(i) = j \\ 0 & \text{otherwise} \end{cases}$$

where *A* is the the number of correctly predicted training examples

- Ants search for solutions by choosing the cutpoint for each variable in a fixed order

Ant Optimisation Representation

- Define a (heuristic) distance to each cutpoint for the next variable:

  $\eta_{ij} =$ accuracy when only using attributes *i*-1 and *i* with cut points *c(i-1)* and *j*

  → large value is more promising

- Otherwise equal to ant systems for the TSP

Experiments

- We employ 2 datasets:
  - The Altman dataset
  - A custom dataset consisting of:
    - 110 firms (55 B and 55 NB)
    - The firms filed for bankruptcy between 1998 and 2004
    - Asset size lower than 1 billion when filing for bankruptcy
    - Using data 2 years prior to bankruptcy
    - The NB set contains firms still ‘alive’ in 2005

Experiments

- The parameters used:
  - $\alpha = 1$
  - $\beta = 1$
  - $\rho = 0.5$
  - 30 ants on the Altman dataset, 40 on the second
  - Different experiments have been performed, using the whole dataset or dividing the latter in a training and test subset.
- Comparison with *multiple discriminant analysis*, used by Altman and the most popular method

Results

type 1 err.	type 2 err.	hitrate	st. dev.	min.	max.
2.8 (8.5%)	1.6 (4.8%)	93.3%	1.06	92.4%	95.5%
2.0 (6.1%)	1.0 (3.0%)	95.5%	N/A	N/A	N/A

TABLE I

RESULTS OBTAINED WITH THE COMPLETE DATA SET 1, THE FIRST ROW USING THE AA, THE SECOND ROW USING MDA.

type 1 err.	type 2 err.	hitrate	st. dev.	min.	max.
11.4 (20.7%)	3.7 (6.7%)	86.3%	0.9	85.5%	87.3%
12.0 (21.8%)	10.0 (18.2%)	80.0%	N/A	N/A	N/A

TABLE III

RESULTS OBTAINED WITH THE COMPLETE DATA SET 2, THE FIRST ROW USING THE AA, THE SECOND ROW USING MDA.

Results

	TRAINING SET	TEST SET
AA	95.2%	90.8%
MDA	97.6%	95.8%

TABLE II

RESULTS WITH DATA SET 1, USING A SEPARATE TRAINING AND TEST SET.

	TRAINING SET	TEST SET
AA	87.1%	81.0%
MDA	77.1%	70.0%

TABLE IV

RESULTS WITH DATA SET 2, USING A SEPARATE TRAINING AND TEST SET.

Results

train 1	test 1	train 2	test 2	train 3	test 3
86.9%	79.5%	88.3%	76.5%	85.7%	76.0%
77.1%	72.7%	81.4%	80.0%	92.9%	72.5%

TABLE V

RESULTS OF THE EXTRA EXPERIMENTS WITH DATA SET 2, THE FIRST ROW WITH THE AA, THE SECOND ROW WITH MDA.